

Yorkshire Durham Geometry Day 2024

Wednesday, 11 December 2024

Upper bounds for Steklov eigenvalues, ratios and gaps - MCS2068 (13:00 - 13:50)

- **Presenter: GITTINS, Katie (Durham University)**

The interplay between the eigenvalues of the Steklov problem and the geometry of the underlying object is a key theme within Spectral Geometry. In the last decade, substantial progress has been made in obtaining geometric upper bounds for the Steklov eigenvalues in various geometric settings. The Steklov spectral ratios and gaps have also received attention in recent years and they too offer fascinating insights into this interplay. In this talk we will first give an overview of some geometric bounds for Steklov eigenvalues. We will then present recent results regarding upper bounds for the Steklov ratios and gaps on balls with revolution-type metrics. The latter is based on joint work with Jade Brisson (Université de Neuchâtel) and Bruno Colbois (Université de Neuchâtel).

Two or infinity - MCS2068 (14:00 - 14:50)

- **Presenter: HRYNIEWICZ, Umberto (RWTH Aachen University)**

The central theme of this talk is an existence result which states that all compact, connected, star-shaped energy levels in a symplectic 4-dimensional vector space admit exactly two or infinitely many periodic orbits. This resolves a conjecture due to Hofer, Wysocki and Zehnder which, in turn, affirmatively resolves the classical conjecture that every Finsler 2-sphere admits exactly two or infinitely many closed geodesics. As a special case, we obtain the famous result of Bangert and Franks, which states that every Riemannian 2-sphere admits an infinite number of closed geodesics. These results are obtained in collaboration with Cristofaro-Gardiner, Hutchings and Liu. If time permits, an extension of the Bangert-Franks result will be discussed which states that the growth of closed geodesics on every Riemannian 2-sphere is at least quadratic with respect to length, thus improving on the known growth rate due to Hingston.

Tea Break - MCS2068 (15:00 - 15:30)

Diffusion geometry in theory and practice - MCS2068 (15:30 - 16:00)

- **Presenter: JONES, Iolo (Durham University)**

In this talk, I will discuss the theory of 'diffusion geometry' and its applications. The heat flow on a manifold encodes a huge amount of its Riemannian geometry. By replacing the heat diffusion operator with a general Markov diffusion operator on a measure space, we can define a 'Riemannian geometry' on a vastly broader class of spaces. This generalises the 'non-smooth differential geometry' of metric measure spaces and also leads directly to a huge family of novel computational tools for geometric data analysis. I will discuss several new applications of diffusion geometry that advance the present state-of-the-art in geometric data analysis and machine learning.

Geodesics, retracts, and the norm-preserving extension property in the symmetrized bidisc - MCS2068 (16:10 - 17:00)

- **Presenter: LYKOVA, Zinaida (Newcastle University)**

A set V in a domain U in \mathbb{C}^n has the norm-preserving extension property if every bounded holomorphic function on V has a holomorphic extension to U with the same supremum norm. We describe all algebraic subsets of the symmetrized bidisc $G = \overline{\text{Irmd}} = \{(z+w, zw) : |z| < 1, |w| < 1\}$ which have the norm-preserving extension property. In contrast to the case of the ball or the bidisc, there are sets in G which have the norm-preserving extension property, but are not holomorphic retracts of G . We give applications to von Neumann-type inequalities for Γ -contractions (that is, commuting pairs of operators for which the closure of G is a spectral set) and for symmetric functions of commuting pairs of contractive operators. The talk is based on joint work with Jim Agler and Nicholas Young. [1] Jim Agler, Zinaida A. Lykova and N. J. Young, Geodesics, retracts, and the norm-preserving extension property in the symmetrized bidisc, *Memoirs of the American Mathematical Society*, 2019, v. 258, no. 1242, 108pp. <https://doi.org/10.1090/memo/1242>