Yorkshire Durham Geometry Day

University of York, Wednesday March 26th 2025

Location. Mathematics Department, Dusa McDuff Room (G/N/135, first floor).

Anyone interested in joining us for lunch should meet us in the Topos at 13:00. We'll take lunch in the Roger Kirk Centre very nearby.

Programme.

14.00 Daniel Platt (Imperial College). *Numerics for 1-forms and Calabi-Yau Manifolds via Neural Networks.*

15.00 Short break.

15.10 Luca Seemungal (Leeds). *The Index of Constant Mean Curvature Surfaces in Three-Manifolds.*

15.40 Sam Engleman (York). *A Morse-Bott cohomology for the moduli space of Higgs bundles.*

16.10 Afternoon Tea.

16.40 Martin Speight (Leeds). L^2 *geometry of vortices.*

There will be a dinner in a restaurant in York in the evening, probably starting at 18.30.

Abstracts.

Daniel Platt (Imperial College). *Numerics for 1-forms and Calabi-Yau Manifolds via Neural Networks.*

Calabi-Yau manifolds are manifolds admitting a unique Ricci-flat metric. Even though existence is known, no explicit formulae for these metrics are known. That frequently causes problems when one wants to compute things that depend on the metric, in particular in Physics. One example in maths is the following: does there exist a harmonic 1-form on a real locus of a Calabi-Yau manifold that is nowhere vanishing? No example is known. In the talk I will explain a conjectural example and an interesting non-example. To define the manifolds, some real algebraic geometry is used. We then numerically (approximately) solve the Ricci-flat equation and the harmonic 1-form equation. It turns out that a neural network is good at that and the approximate solution is easily interpretable. This is based on arXiv:2405.19402, which is joint work with Michael Douglas and Yidi Qi. Time permitting, I will comment on ongoing efforts to turn this into a numerically verified proof that there is a genuine solution near our approximate solution.

Lua Seemungal (Leeds). *The Index of Constant Mean Curvature Surfaces in Three-Manifolds.*

Constant mean curvature (CMC) surfaces are special geometric variational objects, closely related to minimal surfaces. The key properties of a CMC surface are its area, mean curvature, genus, and index. The index of a CMC surface measures its stability: the index counts how many ways one can perturb the surface to decrease the area while keeping the enclosed volume constant. In this talk we discuss relationships between these key properties. In particular we present recent joint work with Ben Sharp, where we bound the index of CMC surfaces linearly from above by genus and the correct scale-invariant quantity involving mean curvature and area.

Local organizers:

York – Ian McIntosh & Graeme Wilkin.

Durham – Fernando Galaz-Garcia & Wilhelm Klingenberg.

Leeds – Derek Harland & Gerasim Kokarev.

Ian McIntosh – 18/02/2025.